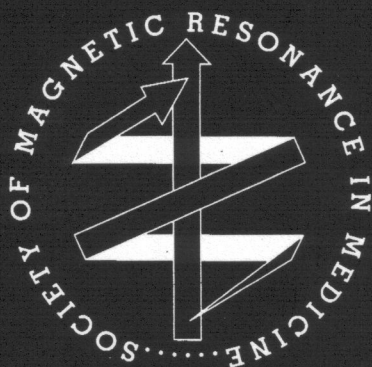


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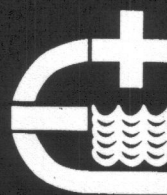
Book of Abstracts

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Diagonal and off-diagonal components of the self-diffusion tensor: their relation to and estimation from the NMR spin-echo signal

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Purpose: We derive an equation relating diagonal and off-diagonal elements of the apparent self-diffusion tensor, \underline{D} , to echo intensity in pulsed-gradient, spin-echo experiments. With it we design pulse sequences to estimate all components of \underline{D} . This procedure is validated by diffusion NMR spectroscopy and imaging of isotropic and anisotropic media. We suggest that errors are made in ignoring off-diagonal elements of \underline{D} in anisotropic diffusion experiments.

Principles: In isotropic media (e.g. water), a scalar self-diffusivity, D , is the constant of proportionality between the gradient in concentration of spin-labeled protons, ∇C , and their flux, \mathbf{J} ; i.e., $\mathbf{J} = -D \nabla C$. Analogously, in anisotropic media (e.g. skeletal muscle or brain white matter), a symmetric second-order apparent self-diffusion tensor, \underline{D} , relates ∇C and \mathbf{J} ; i.e., $\mathbf{J} = -\underline{D} \nabla C$. Diagonal elements of \underline{D} scale fluxes and concentration gradients in the same direction, while off-diagonal elements couple fluxes and concentration gradients in orthogonal directions. The importance of these off-diagonal elements has not been appreciated, nor have they ever been measured.

Theory: Following Stejskal [1], magnetic field gradients and their integrals are defined as:

$$\mathbf{G}(t) = (G_x(t), G_y(t), G_z(t))^T; \mathbf{F}(t) = \int_0^t \mathbf{G}(t') dt'. \quad (1)$$

The echo attenuation by diffusion, $A(TE)/A(0)$, is [1]:

$$\ln\left(\frac{A(TE)}{A(0)}\right) = -\gamma^2 \int_0^{TE} (\mathbf{F}(t') - 2\xi(t')\mathbf{f})^T \underline{D} (\mathbf{F}(t') - 2\xi(t')\mathbf{f}) dt' \quad (2)$$

where γ is the proton gyromagnetic ratio; $\xi(t')$ is the Heaviside function, $H(t' - TE/2)$; and $\mathbf{f} = \mathbf{F}(TE/2)$. When \underline{D} is independent of time, Eq. (2) reduces to:

$$\ln\left(\frac{A(TE)}{A(0)}\right) = -\sum_{i=1}^3 \sum_{j=1}^3 b_{ij} D_{ij}, \quad (3)$$

where the b_{ij} that are analogs to scalar b-factors [2], are calculated numerically or analytically for each sequence using Eq. (2). The b matrix is not necessarily symmetric.

Eq. (3) linearly relates the logarithm of the signal attenuation and each component of \underline{D} . We use multivariate linear regression (with weighted variances) to estimate optimally all components of \underline{D} from measured echo intensities that are produced by field gradients applied in different directions.

Materials and Methods: Diffusion spectroscopy and imaging of water and pork loin samples were performed with a surface coil in a 4.7 T Spectrometer-Imager (GE Omega). Pulsed-gradient

spin-echo sequences, incorporating symmetric trapezoidal gradient pulses ($TR=15$ s; $TE=40$ ms; pulse duration=4.0 ms; rise time=0.2 ms; pulse separation=22.5 ms), were applied in seven non-collinear directions: $(G_x, G_y, G_z) = \{(0, 0, 1), (0, 1, 0), (0, 0, 1), (1, 0, 1), (1, 1, 0), (0, 1, 1), \text{ and } (1, 1, 1)\}$. In each direction, three trials were performed in which gradient strength was increased from 1 to 14 or 15 G/cm in 1-G/cm increments. The total number of acquisitions, N , was either 294 or 315.

Results: For water, the estimated $\underline{D}^{iso} \pm \text{S.E.}$ ($\rho^2 = 0.999998$; $N = 315$) at 14.0°C is:

$$\underline{D}^{iso} = (1.687 \pm 0.0020) \times 10^{-5} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \frac{\text{cm}^2}{\text{sec}}. \quad (4)$$

The estimated $\underline{D}^{0^\circ} \pm \text{S.E.}$ (cm^2/sec) for a pork loin sample at 14.5°C , whose grain was oriented nearly parallel to the x axis, ($\rho^2 = 0.999999$; $N = 294$) is:

$$\underline{D}^{0^\circ} = \begin{pmatrix} 10.137 & 0.365 & -0.530 \\ 0.365 & 9.401 & 0.203 \\ -0.530 & 0.203 & 8.840 \end{pmatrix} \pm \begin{pmatrix} 0.008 & 0.007 & 0.006 \\ 0.007 & 0.008 & 0.006 \\ 0.006 & 0.006 & 0.008 \end{pmatrix} 10^{-6}. \quad (5)$$

The estimated $\underline{D}^{41^\circ} \pm \text{S.E.}$ (cm^2/sec) for the same pork loin sample at 15.0°C , rotated 41° off the x axis in the x - z plane, ($\rho^2 = 0.999999$; $N = 294$) is:

$$\underline{D}^{41^\circ} = \begin{pmatrix} 9.188 & -0.099 & -0.618 \\ -0.099 & 9.346 & 0.038 \\ -0.618 & 0.038 & 9.694 \end{pmatrix} \pm \begin{pmatrix} 0.009 & 0.007 & 0.007 \\ 0.007 & 0.009 & 0.007 \\ 0.007 & 0.007 & 0.009 \end{pmatrix} 10^{-6}. \quad (6)$$

Discussion/Conclusion: The control experiment validates the method to estimate \underline{D} . Statistically significant differences among diagonal components of \underline{D} demonstrate diffusion anisotropy in the pork loin sample. Small S.E. and $\rho^2 \approx 1$ show the multivariate linear model (Eq. (3)) fits the data faithfully; \underline{D} is estimated with high significance.

In anisotropic diffusion, off-diagonal components of \underline{D} vanish only when the "fiber" and "laboratory" frames of reference are coincident [3] - a condition which is rarely verifiable or satisfied. So, diagonal and (non-vanishing) off-diagonal elements of both b and \underline{D} are assumed to affect the measured echo attenuation. As a corollary, at least six experiments are generally required to estimate six independent components of \underline{D} in order to infer microscopic displacements of protons or tissue microstructure [3]. Omitting off-diagonal components of \underline{D} in describing diffusion in anisotropic media also precludes determination of fiber orientation [3].

References:

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